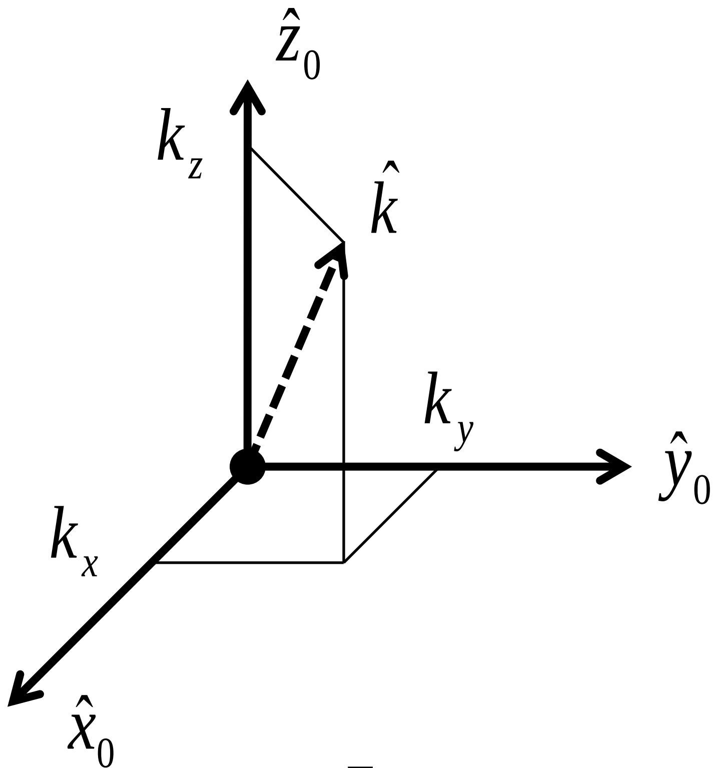
# Day 05

## Rigid Body Transformations

# Rotation About a Unit Axis




$$c_\theta = \cos\theta$$

$$s_\theta = \sin\theta$$

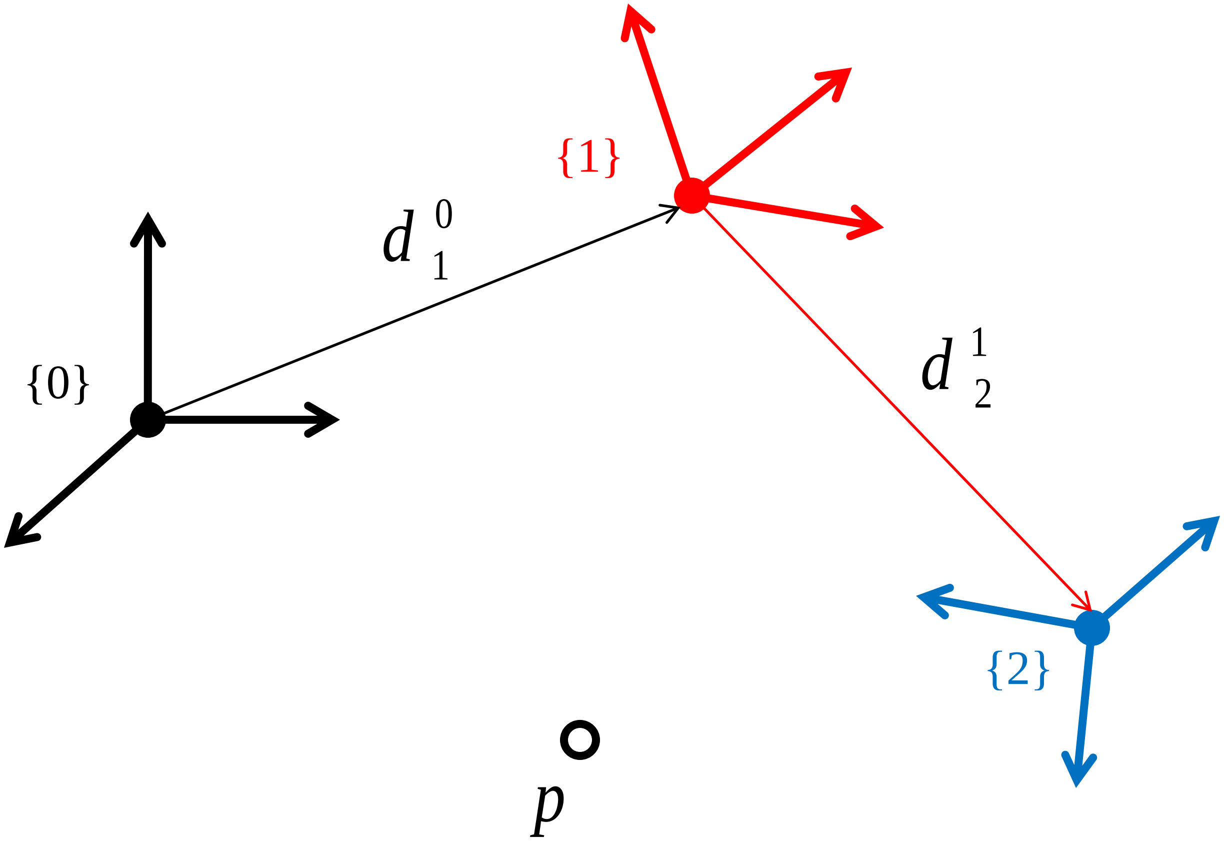
$$v_\theta = 1 - \cos\theta$$

$$R_{k,\theta} = \begin{bmatrix} k_x^2 v_\theta + c_\theta & k_x k_y v_\theta - k_z s_\theta & k_x k_z v_\theta + k_y s_\theta \\ k_x k_y v_\theta + k_z s_\theta & k_y^2 v_\theta + c_\theta & k_y k_z v_\theta - k_x s_\theta \\ k_x k_z v_\theta - k_y s_\theta & k_y k_z v_\theta + k_x s_\theta & k_z^2 v_\theta + c_\theta \end{bmatrix}$$

# Properties of Rotation Matrices

- $R^T = R^{-1}$
- the columns of $R$ are mutually orthogonal
- each column of $R$ is a unit vector
- $\det R = 1$ (the determinant is equal to 1)

# Rigid Body Transformations in 3D



# Homogeneous Representation

- every rigid-body transformation can be represented as a rotation followed by a translation *in the same frame*
  - as a 4x4 matrix

$$T = \begin{bmatrix} & R & & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $R$ is a 3x3 rotation matrix and $d$ is a 3x1 translation vector

# Homogeneous Representation

- in some frame *i*
  - points

$$P^i = \begin{bmatrix} p^i \\ 1 \end{bmatrix}$$

  - vectors

$$V^i = \begin{bmatrix} v^i \\ 0 \end{bmatrix}$$

# Inverse Transformation

- the inverse of a transformation undoes the original transformation
  - if

$$T = \begin{bmatrix} & R & & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - then

$$T^{-1} = \begin{bmatrix} & R^T & & -R^T d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$